

# Effect of charged particle's multiplicity fluctuations on flow harmonics in even-by-event hydrodynamics

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In nucleon-nucleon collisions, charged particle's multiplicity fluctuates. We have studied the effect of multiplicity fluctuation on flow harmonics in nucleus-nucleus collision in event-by-event hydrodynamics. Assuming that the charged particle's multiplicity fluctuations are governed by the negative binomial distribution, the Monte-Carlo Glauber model of initial condition is generalised to include the fluctuations. Explicit simulations with the generalised Monte-Carlo Glauber model initial conditions indicate that the multiplicity fluctuations do not have large effect on the flow harmonics.

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## I. INTRODUCTION

In recent years, there is much interest in event-by-event hydrodynamics. Event-by-event hydrodynamics takes into account that in nucleus-nucleus collisions, participant nucleon positions fluctuates from event to event. Effect of participant nucleon position fluctuations is most prominent on the azimuthal distribution of the produced particles. In a non-zero impact parameter collision between two identical nuclei, the collision zone is asymmetric. Multiple collisions transform the initial asymmetry into momentum anisotropy. Momentum anisotropy is best studied by decomposing it in a Fourier series,

$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left[ 1 + 2 \sum_n v_n \cos(n\phi - n\psi_n) \right], n = 1, 2, 3... \quad (1)$$

$\phi$  is the azimuthal angle of the detected particle and  $\psi_n$  is the plane of the symmetry of initial collision zone. For smooth initial matter distribution (obtained from geometric overlap of density distributions of colliding nuclei), plane of symmetry of the collision zone coincides with the reaction plane (the plane containing the impact parameter and the beam axis),  $\psi_n \equiv \Psi_{RP}, \forall n$ . The odd Fourier coefficients are zero by symmetry. However, fluctuations in the positions of the participating nucleons can lead to non-smooth density distribution, which will fluctuate on event-by-event basis. The participating nucleons then determine the symmetry plane ( $\psi_{PP}$ ), which fluctuate around the reaction plane [1]. As a result odd harmonics, which were exactly zero for smoothed initial distribution, can be developed. It has been conjectured that third harmonic  $v_3$ , which is response of the initial triangularity of the medium, is responsible for the observed structures in two particle correlation in Au+Au

collisions [2],[3],[4],[5],[6],[7]. The ridge structure in  $p\bar{p}$  collisions also has a natural explanation if odd harmonic flows develop. Recently, ALICE collaboration has observed odd harmonic flows in Pb+Pb collisions [8]. In most central collisions, the elliptic flow ( $v_2$ ) and triangular flow ( $v_3$ ) are of similar magnitude. In peripheral collisions however, elliptic flow dominates.

It is well known from experiments that in nucleon-nucleon collisions, charged particles multiplicity fluctuates strongly. Experimental data in limited phase space as well as in full phase space are well described by the negative binomial distribution [9],[10]. Recently, in [11], in a model calculation with Color Glass Condensate (CGC) initial energy density distribution, it was observed that the experimental multiplicity distribution in d+Au collisions at RHIC are better explained if multiplicity fluctuations in NN collisions are included. It was also shown that various moments of the eccentricity of the collision zone in nucleus-nucleus collisions are affected by the multiplicity fluctuations in NN collisions. Fluctuation dominated moments, e.g. triangularity, increase substantially when the effect of multiplicity fluctuations is included. Effect of multiplicity fluctuations on flow coefficients however is not studied in [11]. If, higher moments of the eccentricity of the collision zone increase substantially due to multiplicity fluctuations, an increase in flow harmonics is expected.

In general, event-by-event hydrodynamics do not account for the multiplicity fluctuations in NN collisions. In the present paper, in a simple model, we incorporate the effect of multiplicity fluctuations in event-by-event hydrodynamics and study its effect on the flow harmonics. The model is substantially different from that used in [11]. While in [11], Color Glass Condensate (CGC) model is used to obtain the initial energy density distribution, in the present model, we have used Monte-Carlo Glauber model to obtain the initial energy density distribution, event-by-event. Explicit simulations indicate that multiplicity fluctuations do not have large effect on the flow harmonics.

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## II. MONTE-CARLO GLAUBER MODEL WITH MULTIPLICITY FLUCTUATIONS

In theoretical simulations of event-by-event hydrodynamics, one generally uses the Monte-Carlo Glauber model to obtain the initial energy density distribution in an event. Details of the Monte-Carlo Glauber model can be found in [12]. In a Monte-Carlo Glauber model, according to the density distribution of the colliding nuclei, two nucleons are randomly chosen. If the transverse separation between them is below a certain distance, they are assumed to interact. Transverse position of the participating nucleons is then known in each event. The positions will fluctuate from event-to-event. If a particular event has  $N_{part}$  participants, participants positions in the transverse plane can be labeled as,  $(x_1, y_1), (x_2, y_2), \dots, (x_{N_{part}}, y_{N_{part}})$ . Energy density distribution in the particular event can be obtained by assuming that each participant deposit energy  $\varepsilon_0$  in the transverse plane,

$$\varepsilon(x, y) \approx \varepsilon_0 \sum_{i=1}^{N_{part}} \delta(x - x_i, y - y_i) \quad (2)$$

Fluid dynamical models require continuous density distribution and discrete distribution as in Eq.2 cannot be evolved in a hydrodynamical model. To use in a hydrodynamic model, the discrete density distribution has to be converted into a smooth energy-density distribution. This can be done by smearing the discrete participating nucleon positions by some smoothing function,  $\delta(x - x_i, y - y_i) \rightarrow g(x - x_i, y - y_i, \zeta_1, \zeta_2, \dots)$ ,  $\zeta_i$  being parameters of the smoothing function  $g$ .

$$\varepsilon(x, y) = \varepsilon_0 \sum_{i=1}^{N_{part}} g(x - x_i, y - y_i, \zeta_1, \zeta_2, \dots) \quad (3)$$

One generally uses a Gaussian smoothing function. However, there can be other choices, e.g. in [13], a Woods-Saxon distribution function was used for the smoothing. In the present simulations, we have used a Gaussian distribution

$$g_{gauss}(x - x_i, y - y_i, \sigma) \propto e^{-\frac{(x-x_i)^2 + (y-y_i)^2}{2\sigma^2}}, \quad (4)$$

of width  $\sigma=0.5$  fm.

As it was mentioned earlier, in experiments, charged particle's multiplicity fluctuates in NN collisions. How we incorporate the effect of multiplicity fluctuations within the framework of Monte-Carlo Glauber model? Genesis of Eq.3 gives a clue. It was observed that in nucleus-nucleus collisions, charged particles rapidity density ( $dN_{ch}/d\eta$ ) have two components, (i) a soft component which is proportional to the number of participants

$N_{part}$ , and (ii) a hard component, which is proportional to the number of binary collisions  $N_{coll}$ ,

$$\frac{dN_{ch}^{AA}}{d\eta} = (1-x) \frac{N_{part}}{2} \frac{dN_{ch}^{pp}}{d\eta} + x N_{coll} \frac{dN_{ch}^{pp}}{d\eta} \quad (5)$$

The hard scattering fraction  $x$  is small ( $\sim 0.1-0.2$ ) in RHIC and LHC energy collisions. If we assume that Eq.5 is valid in all the transverse positions  $r_\perp$ , and further assume that initial transverse energy density is proportional to charged particles multiplicity, one can write the transverse energy density distribution, in nucleus-nucleus as,

$$\varepsilon(x, y) = \varepsilon_0 \left[ (1-x) \frac{N_{part}(x, y)}{2} + x N_{coll}(x, y) \right] \quad (6)$$

A large number of experimental data in  $\sqrt{s_{NN}}=200$  GeV Au+Au collisions are well explained in (smooth) hydrodynamic model with initial energy density as given in Eq.6. One understand that the Monte-Carlo Glauber model initial energy density distribution in Eq.3 is essentially a generalisation of Eq.6 with the hard scattering fraction  $x=0$  and  $\varepsilon_0$  in Eq.3 is essentially a measure of charged particles multiplicity in NN collisions.

Charged particle's multiplicity fluctuations in NN collisions imply that even in a single event, all the  $N_{part}$  participant pairs do not produce the same multiplicity i.e.,  $\varepsilon_0$  fluctuates. Considering that in NN collisions, multiplicity fluctuations follow the negative binomial distribution, Monte-Carlo Glauber model of initial condition can be generalised to include multiplicity fluctuations as,

$$\varepsilon(x, y) = \mathcal{N} \sum_{i=1}^{N_{part}} n_i g(x - x_i, y - y_i, \zeta_1, \zeta_2, \dots), \quad (7)$$

In Eq.7,  $(x_i, y_i)$  are the transverse positions of the participant nucleons,  $g$  is a smoothing functions with parameters  $\zeta_i$ .  $n_i$  is a random number following the two parameter negative binomial distribution  $P(n : \langle n \rangle, k)$ .

$$P(n : \langle n \rangle, k) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} \left[ \frac{\langle n \rangle}{\langle n \rangle + k} \right]^n \left[ \frac{k}{\langle n \rangle + k} \right]^k, \quad (8)$$

The parameters  $\langle n \rangle$  and  $k$  corresponds to the average of the distribution and the width of the distribution.  $\mathcal{N}$  in Eq.7 is an overall normalising factor. It can be fixed to reproduce experimental observable, e.g. multiplicity or transverse momentum distribution.

Negative binomial distribution parameters  $\langle n \rangle$  and  $k$  in Eq.8 corresponds to pp collisions multiplicity distributions. However, it is possible that in nucleus-nucleus collisions, pp collision characteristic is changed. For example, in [11], it was shown that experimental data on charged particles multiplicity distribution in d+Au collisions are better fitted with,

$$k = k_{pp} \cdot \min(T_A(r_\perp), T_B(r_\perp))\sigma_0 \quad (9)$$

In the following, we have simulated 20-30% Au+Au collisions at RHIC energy,  $\sqrt{s_{NN}}=200$  GeV. In  $\sqrt{s_{NN}}=200$  GeV Au+Au collisions, average charged particles multiplicity per participant  $\frac{1}{.5N_{part}} \frac{dN_{ch}}{d\eta}$  slowly increase with collision centrality. In 20-30% collision,  $\frac{1}{.5N_{part}} \frac{dN_{ch}}{d\eta} \approx 3.3$  [21]. We then fix  $\langle n \rangle = 3$ . Simulation results for a higher average multiplicity  $\langle n \rangle = 10$  will also be shown. For the width parameter, we have considered two possibilities, which we call model I and II for  $N_{ch}$  fluctuations.

$$\text{Model - I} : k = k_{pp} \cdot \min(T_A(r_\perp), T_B(r_\perp))\sigma_0 \quad (10)$$

$$\text{Model - II} : k = k_{pp} \quad (11)$$

In  $\sqrt{s}=540$  GeV pp collisions, in central rapidity,  $k_{pp} \approx 2$  [10].  $k_{pp}$  decreases logarithmically with energy. We then fix  $k_{pp}=2$ . The difference between model I and II is the following: in model II,  $n_i$  in Eq.7 is a random number following a negative binomial distribution with parameters,  $\langle n \rangle = 3$  and  $k = k_{pp} = 2$ . In model I,  $n_i$  is still a random number following negative binomial distribution with average  $\langle n \rangle = 3$ , but now  $k$  is not a fixed parameter, rather it depend on the participant nucleon positions. Here, I briefly mention the essential difference of the present model with that in [11]. In [11], initial energy density was obtained from a color glass condensate model. In CGC model initial conditions, initial energy density is assumed to be proportional to gluon multiplicity,

$$\frac{dN^{A+B \rightarrow g}}{dy d^2r_T} = K \frac{N_c}{N_c - 1} \int \frac{d^2p_T}{p_T^2} \int_T^p d^2k_T \alpha_x(Q) \quad (12)$$

$$\times \Phi\left(\frac{|\mathbf{p}_T + \mathbf{k}_T|}{2}, x_1\right) \Phi\left(\frac{|\mathbf{p}_T - \mathbf{k}_T|}{2}, x_2\right)$$

where  $\Phi(k_T, x)$  is the unintegrated gluon distribution function. Above equation is interpreted as average local multiplicity. In each cell  $\nabla^2 r_T$ , of the transverse plane, the actual multiplicity is a Negative Binomial distribution random variable. In other words, in [11], in each fluid cell of the transverse plane is associated with fluctuations. In the present model, fluctuations are associated with the participant nucleon only.

### III. HYDRODYNAMIC EQUATIONS, EQUATION OF STATE, INITIAL CONDITIONS

With generalised Monte-Carlo Glauber model initial condition for initial energy density, space-time evolution of the fluid, in each event is obtained by solving the energy-momentum conservation equations,

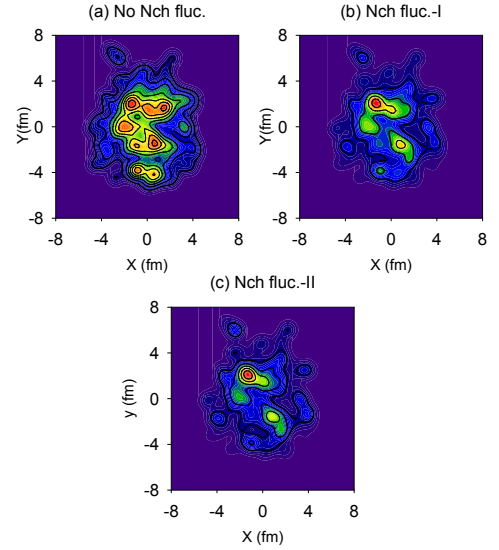


FIG. 1: (color online) Energy density distribution in a typical Monte-Carlo Glauber model event in 20-30% Au+Au collisions. (a) without any multiplicity fluctuations, (b)-(c) with multiplicity fluctuations following model-I and II respectively.

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - Pg^{\mu\nu}, \quad (13)$$

$$\partial_\mu T^{\mu\nu} = 0, \quad (14)$$

where  $\varepsilon$  and  $p$  are the energy density and pressure respectively,  $u$  is the hydrodynamic 4-velocity. We have disregarded any dissipative effect. Assuming boost-invariance, hydrodynamic equations are solved in  $(\tau = \sqrt{t^2 - z^2}, x, y, \eta_s = \frac{1}{2} \ln \frac{t+z}{t-z})$  coordinate system, with the code AZHYDRO-KOLKATA [14]. Hydrodynamics equations are closed with an equation of state (EoS)  $p = p(\varepsilon)$ . Currently, there is consensus that the confinement-deconfinement transition is a cross over. The cross over or the pseudo critical temperature for the transition is  $T_c \approx 170$  MeV [15–18]. In the present study, we use an equation of state where the Wuppertal-Budapest [15, 17] lattice simulations for the deconfined phase is smoothly joined at  $T = T_c = 174$  MeV, with hadronic resonance gas EoS comprising all the resonances below mass  $m_{res}=2.5$  GeV. Details of the EoS can be found in [19].

In addition to the initial energy density for which we use the Monte-Carlo Glauber model, with or without multiplicity fluctuations, solution of hydrodynamic equations requires to specify the thermalisation or the initial time  $\tau_i$  and fluid velocity  $(v_x(x, y), v_y(x, y))$  at the initial time. A freeze-out prescription is also needed to convert the information about fluid energy density and velocity to invariant particle distribution. We assume that the fluid is thermalized at  $\tau_i=0.6$  fm and the initial fluid velocity is zero,  $v_x(x, y) = v_y(x, y) = 0$ . The freeze-out is fixed at  $T_F=130$  MeV. We use Cooper-Frye formalism to obtain the invariant particle distribution of  $\pi^-$  from the freeze-out surface. Resonance production is in-

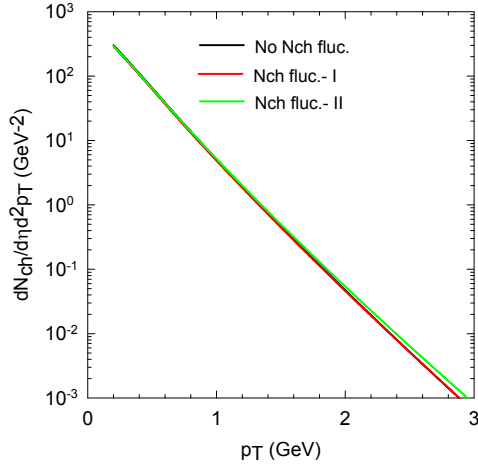


FIG. 2: (color online) Charged particles transverse momentum distribution with and without multiplicity fluctuations.

cluded. Considering that pions constitute  $\sim 20\%$  of all the charged particles,  $\pi^-$  invariant distribution is multiplied by the factor  $2 \times 1.2$  to approximate the charged particle's invariant distribution. From the invariant distribution, harmonic flow coefficients are obtained as [20],

$$v_n(y, p_T) e^{in\psi_n(y, p_T)} = \frac{\int d\phi e^{in\phi} \frac{dN}{dy p_T dp_T d\phi}}{\frac{dN}{dy p_T dp_T}} \quad (15)$$

$$v_n(y) e^{in\psi_n(y)} = \frac{\int p_T dp_T d\phi e^{in\phi} \frac{dN}{dy p_T dp_T d\phi}}{\frac{dN}{dy}} \quad (16)$$

In a boost-invariant version of hydrodynamics, flow coefficients are rapidity independent. Present simulations are suitable only for central rapidity,  $y \approx 0$ , where boost-invariance is most justified. Hereafter, we drop the rapidity dependence.  $\psi_n$  in Eqs.15,16 is the participant plane angle for the  $n$ -th flow harmonic. We characterise the asymmetry of the initial collision zone in terms of various moments of the eccentricity [5],[6],[7],

$$\epsilon_n e^{in\psi_n} = - \frac{\int \int \epsilon(x, y) r^n e^{i2\phi} dx dy}{\int \int \epsilon(x, y) r^n dx dy}, n = 2, 3, 4, 5 \quad (17)$$

$\epsilon_2$  and  $\epsilon_3$  are called eccentricity and triangularity. In the following,  $\epsilon_4$  and  $\epsilon_5$  will be called rectangularity and penta-angularity respectively. They measure the square-ness and five sided-ness of the initial distribution respectively. Eq.17 also determine the participant plane angle  $\psi_n$ .

#### IV. RESULTS

To study the effect of multiplicity fluctuations in event-by-event hydrodynamics, we have simulated 20-30% Au+Au collisions for  $N_{event}=100$  events. Recently

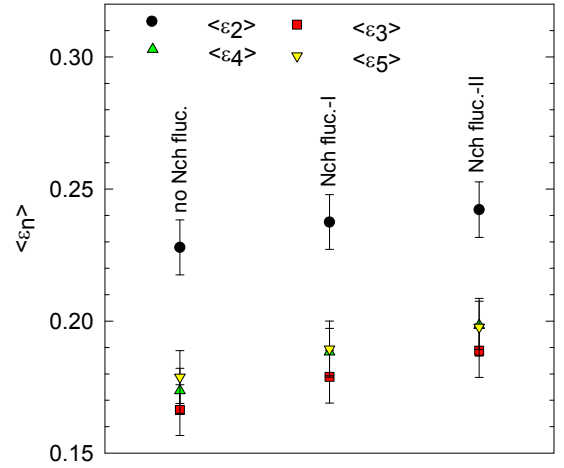


FIG. 3: (color online) Simulation results for event averaged asymmetry parameters  $\epsilon_n$ ,  $n=2-4$ .  $\epsilon_n$  marginally increase when multiplicity fluctuations are included.

in [22], it was shown that with fluctuating initial conditions, event averaged as well as variance of elliptic flow and triangular flow remain approximately unaltered for  $N_{event}=50-2500$ .  $N_{event} = 100$  is then sufficiently large to study the effect of multiplicity fluctuations in event-by-event hydrodynamics. In each event, Monte-Carlo Glauber model participant positions are smoothed with a Gaussian function of width  $\sigma=0.5$  fm. Multiplicity fluctuations are accounted either with model I or model II. We also simulate events without any multiplicity fluctuations. The overall normalising factor  $\mathcal{N}$  is fixed such that event averaged charged particles multiplicity reproduces the experimental value,  $\frac{dN_{ch}}{dy} \approx 276$  in 20-30% Au+Au collisions.

In Fig.1, effect of multiplicity fluctuations on the initial energy density distribution, in a typical Monte-Carlo event is shown. Panel (a) shows the energy density distribution without any multiplicity fluctuations. Energy density distributions with multiplicity fluctuations are shown in panel (b) and (c) respectively for model-I and II. Effect of multiplicity fluctuations is to smooth the energy density distribution. The fine structures which are seen in panel (a) get diffused when multiplicity fluctuations are included.

##### A. Transverse momentum spectra

In Fig.2, simulation results for the charged particle's transverse momentum distribution are shown. The lines represent the event averaged values. The standard errors are small and are not shown here. The black lines in Fig.2, are the simulated spectra when multiplicity fluctuations are neglected. The red and green lines are simulated spectra with multiplicity fluctuations included following model-I and II respectively. Transverse momentum distribution is marginally affected due to multiplicity

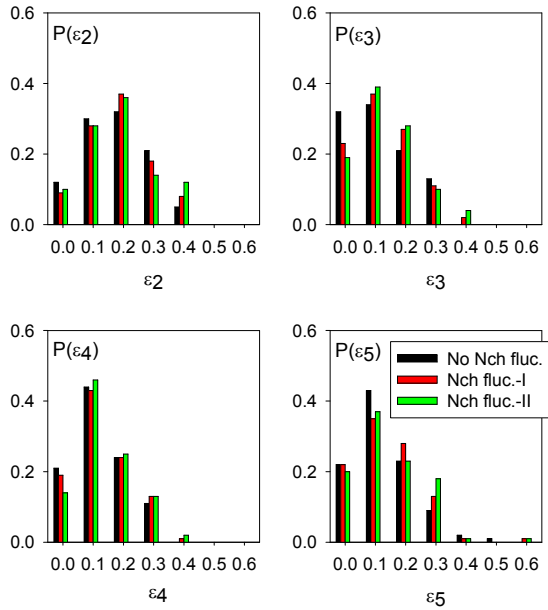


FIG. 4: (color online) Probability distribution of initial spatial anisotropy  $P(\epsilon_2)$ ,  $P(\epsilon_3)$ ,  $P(\epsilon_3)$ ,  $P(\epsilon_4)$  and  $P(\epsilon_5)$  in simulated events are shown. In each panel, the black colors represent simulations without multiplicity fluctuations. Multiplicity fluctuations with model I and II are indicated by red and green colors.

ity fluctuations. Indeed, in the figure, one can not distinguish between spectra obtained without multiplicity fluctuations and with multiplicity fluctuations I. For multiplicity fluctuations II, the spectrum is marginally hardened.

### B. Integrated flow coefficients

In a hydrodynamical model, development of the flow coefficients depend on the initial spatial anisotropy parameters ( $\epsilon_n$ ) characterising the collision zone. Approximately, larger the initial asymmetry, more is the flow. Does the initial asymmetry parameters  $\epsilon_n$  changes when multiplicity fluctuations are included in the Monte-Carlo Glauber model of initial condition? Simulation results for the event averaged asymmetry parameters  $\epsilon_n$  and its standard errors are shown in Fig.3.  $\langle \epsilon_n \rangle$  increase marginally when multiplicity fluctuations are included for. The increase is barely more in model-II than in model I. Marginal effect of multiplicity fluctuations on asymmetry parameter is also evident in Fig.4, where, in four panels, histograms for the probability distribution of the initial eccentricity ( $\epsilon_2$ ), triangularity ( $\epsilon_3$ ), rectangularity ( $\epsilon_4$ ) and penta-angularity ( $\epsilon_5$ ), in the simulated events are shown. The black bars correspond to simulations without multiplicity fluctuations. The red and green bars are obtained when multiplicity fluctuations are included according to model I and II respectively. Qualitatively, probability distribution of asymmetry pa-

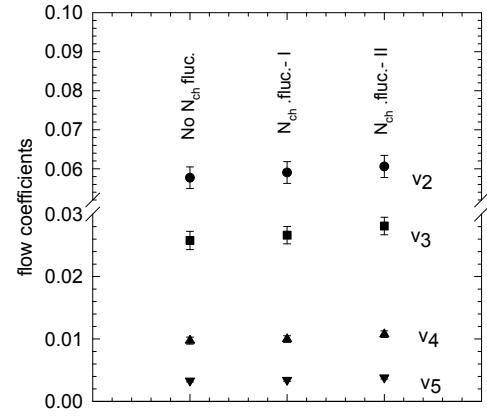


FIG. 5: Simulated flow coefficients  $v_n$ ,  $n=2,3,4$  and 5 with and without energy fluctuations.

rameter  $\epsilon_n$  remain similar. The result is not unexpected. In Monte-Carlo Glauber model, the asymmetry parameters are determined mainly by the positions of the participant nucleons (see Eq.17), weighted by the energy density. Since the participant positions remain unaltered the change in  $\epsilon_n$  due to multiplicity fluctuations is small.

In Fig.5, simulated (integrated) flows,  $v_n$ ,  $n=2-5$ , with and without the multiplicity fluctuations are shown. The symbols represent the event averaged values, the bars the standard errors. Event averaged flow coefficients marginally increase due to multiplicity fluctuations. However, the increase is small. In model I of fluctuations, flow coefficients  $v_n$ ,  $n=2-5$  increase by less than 5%. In model II also, the increase is modest, less than 5% in  $v_2$  and  $v_3$ . In higher flow coefficients  $v_4$  and  $v_5$ , the increase is marginally more,  $\sim 10-15\%$ . The result is in agreement with our observations that initial asymmetry parameters depend marginally on the multiplicity fluctuations in NN collisions.

Recently, in [23], correlation between elliptic ( $v_2$ ) and triangular ( $v_3$ ) flow with initial eccentricity ( $\epsilon_2$ ) and initial triangularity ( $\epsilon_3$ ) was studied. It was shown that in event-by-event hydrodynamics, elliptic flow is strongly correlated with initial eccentricity. Comparatively weak correlation was observed between triangular flow and initial triangularity. Correlation between higher flow coefficients,  $v_4$ ,  $v_5$  and initial spatial asymmetry parameter  $\epsilon_4$ ,  $\epsilon_5$  were not studied. It is interesting to study the correlation between higher flow coefficients  $v_4$ ,  $v_5$  and asymmetry parameters  $\epsilon_4$  and  $\epsilon_5$ . Effect of multiplicity fluctuations on the correlation between different flow coefficients and asymmetry measures, is also interesting. In Fig.6, simulated flow coefficients  $v_n$  are plotted against the initial asymmetry parameters  $\epsilon_n$ . Simulation results with and without multiplicity fluctuations are shown in separate panels. One observes that elliptic flow  $v_2$  is strongly correlated with initial eccentricity  $\epsilon_2$ . The triangular flow  $v_3$  also appear to be correlated with initial triangularity, though the degree of correlation is less than in elliptic flow. However, correlation between higher flow

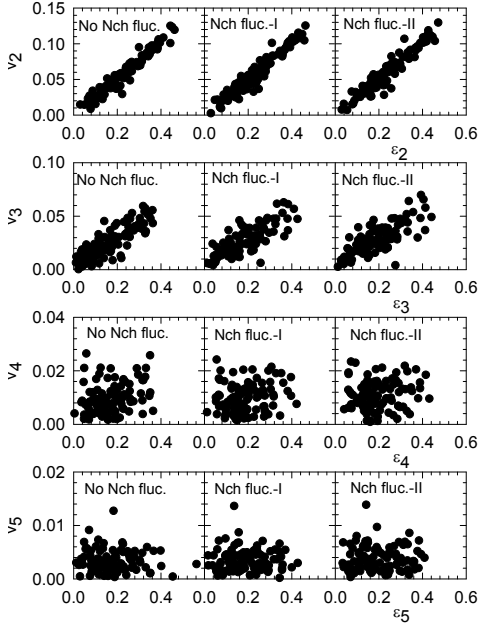


FIG. 6: Effect of multiplicity fluctuations on the correlation between flow coefficients and initial asymmetry parameters,  $(v_2, \epsilon_2)$ ,  $(v_3, \epsilon_3)$ ,  $(v_4, \epsilon_4)$  and  $(v_5, \epsilon_5)$ , are shown.

coefficients  $v_4$  and  $v_5$  with spatial asymmetry  $\epsilon_4$  and  $\epsilon_5$  appears to be weak. Effect of multiplicity fluctuations on the correlation appear to be marginal again. In [23] a quantitative measure was defined to quantify the correlation between flow coefficients and initial spatial asymmetry measure. A modified form is used here to measure the correlation,

TABLE I: Correlation measure for flow harmonics  $v_n$ ,  $n=2-5$  in the event-by-event hydrodynamics with and without multiplicity fluctuations.

	$C_{measure}$			
	$v_2$	$v_3$	$v_4$	$v_5$
no Nch fluc.	0.980	0.873	0.633	0.712
Nch fluc.-I	0.974	0.865	0.538	0.727
Nch fluc.-II	0.971	0.869	0.466	0.689

$$C_{measure}(n) = 1 - \frac{\sum_i [v_n^i(\epsilon_n) - v_{n,st.line}(\epsilon_n)]^2}{\sum_i [v_n^i(\epsilon) - v_{st.line}(\epsilon)]^2} \quad (18)$$

$C_{measure}$  essentially measures the dispersion of the simulated flow coefficients from the best fitted straight line, relative to completely random flow coefficients. It varies between 0 and 1. If flow coefficients are perfectly correlated then  $v_n \propto \epsilon_n$  and  $C_{measure}$  is identically unity. For completely random flow coefficients,  $C_{measure}=0$ . To obtain an even ground for comparison of  $C_{measure}$  for different flow coefficients, the flow coefficients ( $v_n$ ) and the

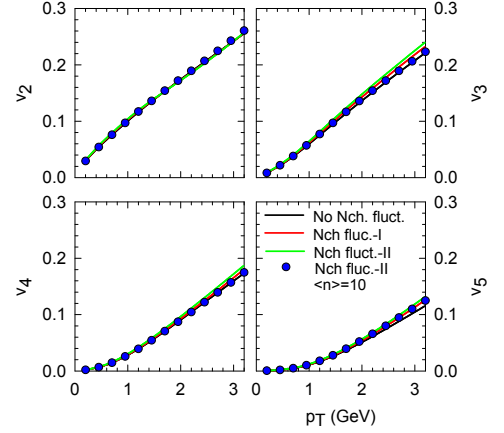


FIG. 7: (color online) Simulated  $v_2$ ,  $v_3$ ,  $v_4$  and  $v_5$  are shown in four panels. In each panel, flow coefficients, without multiplicity fluctuations (the black line), with multiplicity fluctuations with model I (the red line) and with model II (the green line) are shown. Average  $\langle n \rangle$  of the negative binomial distribution is fixed at  $\langle n \rangle=3$ . The blue circles are simulation results with model II of fluctuations but with  $\langle n \rangle=10$ .

asymmetry parameters ( $\epsilon_n$ ) are scaled to vary between 0 and 1. In table.I, we have listed the  $C_{measure}$  values obtained in the present simulations.

Let us first discuss the correlation measures without any multiplicity fluctuations. Elliptic flow is strongly correlated with initial eccentricity,  $C_{measure}(v_2) \sim 0.98$ . Correlation in triangular flow is comparatively weak,  $C_{measure}(v_3) \sim 0.87$ . Correlation between higher flow harmonics,  $v_4$  and the initial asymmetry parameter  $\epsilon_4$  or between  $v_5$  and  $\epsilon_5$  is much more weaker than that in elliptic or triangular flow,  $C_{measure}(v_4) \sim 0.63$  and  $C_{measure}(v_5) \sim 0.71$ . If departure of  $C_{measure}$  from unity is a measure of flow uncorrelated with the initial eccentricity measure, only  $\sim 2\%$  of elliptic flow is uncorrelated with initial eccentricity. Uncorrelated flow is more  $\sim 13\%$  for triangular flow. For higher harmonics,  $v_4$  and  $v_5$ , uncorrelated flow is much more,  $\sim 40\%$  in  $v_4$  and  $30\%$  in  $v_5$ . Multiplicity fluctuations appear to reduce the correlation between flow coefficients and asymmetry measures. However, with the exception of  $v_4$ , correlation between flow coefficients and asymmetry measures are reduced only marginally (less than a few percent). Only in  $v_4$ , correlation is significantly reduced, e.g. by  $\sim 15\%$  in model I of fluctuations and by  $\sim 26\%$  in model II of fluctuations. We conclude that with the exception of flow harmonic  $v_4$ , the correlation between flow coefficients and initial asymmetry parameter remain largely unaffected by the inclusion of multiplicity fluctuations in NN collisions. Why the correlation between flow harmonic  $v_4$  and the asymmetry measure  $\epsilon_4$  is particularly sensitive to multiplicity fluctuations is not understood. More study is required to resolve the issue.



### C. Differential flow coefficients

As it is with the integrated flow coefficients, differential flow coefficients also depend marginally on multiplicity fluctuations. In Fig.7, in four panels, simulation results for the differential flow coefficients,  $v_2(p_T)$ ,  $v_3(p_T)$ ,  $v_4(p_T)$  and  $v_5(p_T)$  are shown. Elliptic flow is hardly modified by inclusion of fluctuations. Detailed inspection indicate that in the  $p_T$  range 1-2 GeV, differential elliptic flow is decreased by less than  $\sim 2\%$  when multiplicity fluctuations are included. In triangular flow, flow is marginally increased due to multiplicity fluctuations. In the  $p_T$  range 1-2 GeV, in model I of fluctuations, the increase is about  $\sim 4\text{-}5\%$ . The increase is marginally more  $\sim 8\%$  in model II. Higher flow coefficients  $v_4$  and  $v_5$  also increase marginally, e.g. by 6-7% and by 10-12% respectively in the  $p_T$  range 1-2 GeV. We conclude that differential flow coefficients  $v_2(p_T)$ - $v_5(p_T)$  are only marginally affected by multiplicity fluctuations in NN collisions.

Negative binomial distribution has two parameters, the width  $k$  and average  $\langle n \rangle$ . In the simulations presented here, two possibilities for the width parameter  $k$  was considered. The average of the distribution was kept fixed at  $\langle n \rangle = 3$ . To check whether or not simulated flows depend sensitively on the average of the distribution, we have simulated 100 events with  $\langle n \rangle = 10$ ,  $k = k_{pp} = 2$ . The results (the blue circles) are shown in Fig.7. Simulated flows with  $\langle n \rangle = 10$  can hardly be distinguished from the simulated flows with  $\langle n \rangle = 3$ .

## V. SUMMARY AND CONCLUSIONS

To summarise, we have studied the effect of multiplicity fluctuations in NN collisions on flow harmonics in

$\sqrt{s_{NN}} = 200$  GeV Au+Au collisions. The Monte-Carlo-Glauber model for the initial transverse energy distribution is generalised to include charged particle's multiplicity fluctuations in NN collisions. Multiplicity fluctuations are assumed to be governed by the negative binomial distribution, with two parameters, the average  $\langle n \rangle$  and the width  $k$ . With the generalised Monte-Carlo-Glauber model initial transverse energy distribution, we have simulated 20-30% Au+Au collisions event-by-event. Two possibilities for the width parameter  $k$  are considered, (i) model I:  $k = k_{pp} \cdot \min(T_A(r_\perp), T_B(r_\perp))\sigma_0$ , i.e. the width depend on the local position of the participant nucleons, and (ii) model II:  $k = k_{pp}$ , i.e. the width remain same as in pp collision. In the simulations, we use  $k_{pp} = 2$ . The average the distribution is kept fixed at  $\langle n \rangle = 3$ . The simulations are constrained to reproduce experimentally charged particles multiplicity in 20-30% collisions. Explicit simulations indicate that in event-by-event hydrodynamics, multiplicity fluctuations do not play significant role in the development of the flow harmonics. Integrated flows are marginally increased with multiplicity fluctuations. Increase is barely more in model II of fluctuations than in model I. The differential flow is also increased marginally. Indeed, within the uncertainties integrated and differential flow coefficients remain largely unaltered irrespective of multiplicity fluctuations. We have also studied the correlation between different flow harmonics and initial asymmetry measures. With the exception of fourth flow harmonic ( $v_4$ ), correlation between flow coefficients and asymmetry measures also remain largely unaltered. We conclude that in Monte-Carlo Glauber model of initial condition, flow harmonics are not largely affected by the charged particle's multiplicity fluctuations in NN collisions.

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